where $\theta = \varepsilon_x + \varepsilon_y + \varepsilon_z$. In uniaxial strain, $\varepsilon_x = (V_0 - V)/V_0$, $\varepsilon_y = \varepsilon_z = 0$. If the equality holds, the material is in the plastic state.

ii) In the plastic state, every increment in strain is the sum of an elastic and a plastic increment:

(43a) $$\mathrm{d}\varepsilon_x = \mathrm{d}\varepsilon_x^e + \mathrm{d}\varepsilon_x^p,$$

(43b) $$\mathrm{d}\varepsilon_y = \mathrm{d}\varepsilon_y^e + \mathrm{d}\varepsilon_y^p,$$

(43c) $$\mathrm{d}\varepsilon_z = \mathrm{d}\varepsilon_z^e + \mathrm{d}\varepsilon_z^p.$$

iii) There is no plastic dilatation:

(44) $$\mathrm{d}\varepsilon_x^p + \mathrm{d}\varepsilon_y^p + \mathrm{d}\varepsilon_z^p = 0.$$

iv) The stress is supported solely by the elastic strain:

(45a) $$\mathrm{d}p_x = \lambda\,\mathrm{d}\theta + 2\mu\,\mathrm{d}\varepsilon_x^e,$$

(45b) $$\mathrm{d}p_y = \lambda\,\mathrm{d}\theta + 2\mu\,\mathrm{d}\varepsilon_y^e,$$

(45c) $$\mathrm{d}p_z = \lambda\,\mathrm{d}\theta + 2\mu\,\mathrm{d}\varepsilon_z^e,$$

where λ and μ are, in general, functions of the density.

As p_x is increased from zero, the response is initially elastic and $\varepsilon_y = \varepsilon_z = 0$. Then

(46) $$p_x - p_y = (1 - 2\nu)p_x/(1 - \nu),$$

where $\nu = \lambda/2(\lambda + \mu)$ is Poisson's ratio. The yield stress is reached at a value of p_x called the «Hugoniot elastic limit», denoted by p_{HEL}. From eqs. (41) and (46):

(47) $$p_{\mathrm{HEL}} = (1 - \nu)Y/(1 - 2\nu).$$

For further increases in p_x, the material is in the plastic state. Then

(48) $$p_x \equiv \bar{p} + \tfrac{2}{3}(p_x - p_y) = \bar{p} + 2Y/3,$$

where $\bar{p} = (p_x + p_y + p_z)/3$, a function of density and internal energy alone. Referring to Fig. 14 b), eq. (48) applies to the segment AB of the p_x curve. The slope of the (p_x, V) curve in the elastic region is, from eqs. (42):

(49) $$\mathrm{d}p_x/\mathrm{d}V = -(\lambda + 2\mu)/V_0 = -(K + 4\mu/3)/V_0,$$

where K is bulk modulus. In the plastic region, AB, the slope is, for constant Y, from eq. (48)

$$\text{(50)} \qquad \mathrm{d}p_x/\mathrm{d}V = \mathrm{d}\overline{p}/\mathrm{d}V = -K/V\,.$$

In accord with eq. (50), it is convenient to define the incremental dilatation as $\mathrm{d}V/V$. Bulk modulus normally increases with $\overline{p}$, so AB is normally concave upward. The yield stress, Y, is in general a function of plastic work and density. In such case eq. (50) is augmented by a $\mathrm{d}Y/\mathrm{d}V$ term. In any case the offset of p_x from the hydrostat, $\overline{p}$, is always $2Y/3$.

At point B in Fig. 14 *b*) we suppose that a change is made from monotonically increasing to monotonically decreasing p_x. Equation (41) must again be examined to determine whether the mass element is in the elastic or plastic state. During the initial compression process, p_x increased more rapidly than p_y until yield occurred. During unloading, p_x decreases more rapidly than p_y until yielding again occurs. Thus the portion BC of the unloading curve is elastic until $p_y - p_x = Y$ at C. From C to D, unloading is plastic and the unloading curve lies below the hydrostat by $\frac{2}{3}Y$.

Referring to the discussion following eq. (17), we see that point A of Fig. 14 *b*) may be a point of instability for single shock compressions. To see that this is indeed the case, suppose that a shock wave has been generated with amplitude p_{HEL}, traveling with speed

$$D_E = [V_0(\lambda + 2\mu)]^{\frac{1}{2}}\,.$$

The velocity of this shock front relative to the material behind it is

$$\text{(51)} \qquad D_e - u_E = (V_A/V_0)D_E = V_A\sqrt{(\lambda + 2\mu)/V_0}\,.$$

If an additional compression of small amplitude is produced to follow the already established shock, it will travel with velocity c_A relative to the material ahead of it, where, according to eq. (50),

$$c_A = \sqrt{KV_A} = V_A\sqrt{(\lambda + 2\mu/3)/V_A}\,.$$

Comparing this with eq. (51) we find that

$$\text{(52)} \qquad (D_E - u_E)^2/c_A^2 = (3V_A/V_0)(1-\nu)/(1+\nu) \simeq 3(1-\nu)/(1+\nu) = \tfrac{3}{2} \quad \text{for } \nu = \tfrac{1}{3}\,,$$

since $V_A/V_0 \simeq 1$ at the Hugoniot elastic limit. According to eq. (52), the second wave does not overtake the shock, so there is a region of the (p_x, V) curve above the point A which cannot be reached by a single shock from